

ECML 2013 Calculus

ESSEX COUNTY MATH LEAGUE

May 22, 2013

Calculus

DIRECTIONS: You **may** write on this test. Be sure that your name, subject, and school (including town name) are on the answer sheet. Mark the answer sheet with dark, careful marks using a #2 pencil. Your score will be determined by the number of correct answers. Incorrect or blank answers will **NOT** lower your score. You **MAY** use only a SAT I approved calculator on this test. The answer to the tie-breaker should be placed on the answer sheet in the place indicated by the proctors. The tie-breaker will be scored, only in the case of a tie between the top scorers, and will not count as part of the team score.

The answer to part e) will always be **NG** for "Not Given". This is a viable answer and means that the correct answer is **not** one of the first four listed.

1) Find: $\lim_{x \rightarrow 0} \left(\frac{x}{2(\sqrt{x+5} - \sqrt{5})} \right) =$

- A) 0 B) $\sqrt{5}$ C) $\sqrt{10}$ D) $2\sqrt{5}$ E) NG

2) Find the coordinates of the points at which the graph $y = 2x^3 - 3x^2 - 12x + 7$ has horizontal tangents.

- A) (2, 13) and (-2, 3) B) (-2, 3) and (1, -6) C) (-1, 14) and (1, -6)
d) (2, -13) and (-1, 14) E) NG

3) What is the equation of the tangent line to $y = x \sin^2(2x)$ at the point $x = \frac{\pi}{4}$?

- A) $y = x$ B) $y = -x$ C) $y = (\pi + 1)x$ D) $y = (-\pi + 1)x$ E) NG

4) On which of the following open intervals is $f(x) = (2x+3)\sqrt{x}$ concave down?

- A) $x > 0$ B) $x > \frac{1}{2}$ C) $0 < x < \frac{1}{2}$ D) $0 < x < 1$ E) NG

5) Find $f(-1)$ if $f(1)=0$, $f'(1)=-2$, and $f''(x)=6x-3$.

- A) $-\frac{5}{2}$ B) -2 C) 2 D) $\frac{5}{2}$ E) NG

6) Find : $\int \frac{\cos(\sqrt{x-1})}{\sqrt{x-1}} dx =$

- A) $\frac{1}{2}\sin(\sqrt{x-1})+C$ B) $2\sin(\sqrt{x-1})+C$ C) $\frac{1}{2}\cos^2(\sqrt{x-1})+C$

- d) $-\sin(\sqrt{x-1})+C$ E) NG

7) Find: $\lim_{x \rightarrow 1} \frac{1 + \cos(\pi x)}{x^2 - e^{2x-2}} =$

- A) $\frac{\pi^2}{2}$ B) $-\frac{\pi^2}{2}$ C) $\frac{\pi}{2}$ D) $-\frac{\pi}{2}$ E) NG

8) Find the slope of the tangent line to $xy^2 - x^2y = 2$ at the point $x = -2$.

- A) -1 B) 0 C) 1 D) undefined E) NG

9) The graph of $y = \frac{x^2 - 4x - 12}{x + 3}$ has which of the following asymptotes?

I $x = -3$

II $y = 1$

III $y = x - 7$

- A) I only B) II and III only C) I and III only

- d) I, II and III E) NG

10) Evaluate: $\int_{-\ln(2)}^{\ln(2)} \frac{1 - 2e^{-x}}{e^x} dx =$

- A) $-\frac{9}{4}$ B) $-\frac{7}{4}$ C) $\frac{7}{4}$ D) $\frac{9}{4}$ E) NG

- 11) A particle is moving along the curve with equation $\frac{x^2 y}{x+y} = \frac{4}{3}$. It is known that the y-coordinate is increasing at the rate of $\frac{1}{2}$ units/sec when the particle is at the point (2, 1). What is the rate in units/sec at which the x-coordinate is changing at that instant?
- A) -5 B) -2 C) 2 D) 5 E) NG
- 12) Find: $\int 4x^3 \ln(x) dx$
- A) $4x^2 + 6 + \frac{4}{x^2}$ B) $4x^2 - 6 + \frac{4}{x^2}$ C) $4x^2 + 6 - \frac{4}{x^2}$ D) $4x^2 - 6 - \frac{4}{x^2}$ E) NG
- 13) When $f(x) = 6 + x - x^2$, for what values of x is the function equal to its average on the interval $0 \leq x \leq 3$?
- A) $\frac{9}{2}$ B) $\frac{9}{4}$ C) $\frac{\sqrt{7}-1}{2}$ D) $\frac{\sqrt{7}+1}{2}$ E) NG
- 14) The water tank for Smallville is a sphere of radius 30 ft on top of a tower. The town engineer reported to the council that, even though they needed to keep the tank at least 60% full, she found that the depth of the water to be only 27 ft. This is approximately what percentage of the full tank?
- A) 42.5% B) 41.5% C) 40.5% D) 39.5% E) NG
- 15) The area between the x-axis and the graph of $y = -\frac{1}{4}(x-2)^2 + 4$ is divided in half by a line which intersects the parabola and the y-axis at the same point. If the line intersects the x-axis at the point (a, 0), find the value of a.
- A) $-\frac{27}{44}$ B) $\frac{22}{9}$ C) $\frac{44}{9}$ D) $\frac{44}{3}$ E) NG

Tie Breaker. This question will only be scored to break a tie amongst the high scorers on this contest. It will not count as part of the team score.

The vertical line $x=k$ divides the area under the curve $y = \sin(2x - \frac{\pi}{2}) + 1$, which lies in the first quadrant in half. Find the value of k to 2 decimal places.